

1. a) numerator ; denominator

b) rational expression

c) zero

d) not allowed ; illegal ; "undefined"

$$2. \quad c) \quad \frac{65}{78} = \frac{13 \times 5}{13 \times 6} = \left(\frac{13}{13} \right) \frac{5}{6}$$

$$\frac{(x+2)(x+1)}{(x+2)(x+3)} = \frac{x+1}{x+3}$$

$$3. \quad a) \quad \frac{3}{\textcircled{x}} \quad x \neq 0$$

$$b) \quad \frac{x+2}{\textcircled{x-1}} \quad \begin{array}{l} x-1 \neq 0 \\ x \neq 1 \end{array}$$

$$c) \quad \frac{x-1}{\textcircled{x^2+1}} \quad \begin{array}{l} x^2+1 \neq 0 \\ x^2 \neq -1 \end{array}$$

no restriction

$$\text{Bonus} \quad \frac{x-1}{\textcircled{x^2-1}} = \frac{x-1}{(x+1)(x-1)} \quad \begin{array}{l} x \neq \pm 1 \\ \pm 1 \\ \uparrow \\ x^2-1 \neq 0 \quad x^2 \neq 1 \quad x = \pm \sqrt{1} \end{array}$$

$$e) \frac{x^2 - x - 6}{x^2 - 4x}$$

$$x^2 - 4x \neq 0$$

$$x(x-4) \neq 0$$

$$x \neq 0, 4$$

$$\boxed{\begin{array}{c} 4-1 \\ + \\ 4 \cdot 2 \end{array}}$$

$$j) \frac{x^2 - 4}{x^4 - 1}$$

$$x^4 - 1 \neq 0$$

$$(x^2 + 1)(x^2 - 1) \neq 0$$

$$(x^2 + 1)(x + 1)(x - 1) \neq 0 \quad x \neq \pm 1$$

$$4. \quad a) \frac{\cancel{x-1}}{\cancel{x-1}} = 1$$

$$c) \frac{x+1}{1+x} \rightarrow \frac{x+1}{x+1} = 1$$

$$b) \frac{x-1}{1-x} = \frac{x-1}{-1(-1+x)} = \frac{\cancel{x-1}}{-1(\cancel{x-1})} = \frac{1}{-1} = -1$$

$$\boxed{\frac{a-b}{b-a} = -1}$$

$$g) \frac{(\overset{-1}{\cancel{x-2}})(\overset{-1}{\cancel{x-1}})(x+2)}{(\cancel{2-x})(\cancel{1-x})(2-x)} = \frac{x+2}{2-x}$$

$$5. \quad e) \quad \frac{x^2 - xy}{x^2} = \frac{\cancel{x} \boxed{x-y}}{\boxed{\cancel{x}} \boxed{x}} = \frac{x-y}{x}$$

$$d) \quad \frac{(x+1)\cancel{(x-1)}}{(x-1)\cancel{(x+1)}} = \frac{x+1}{x-1}$$

$$t) \quad \frac{x^2 - 3xy + 2y^2}{x^2 - 4y^2} = \frac{\cancel{(x-2y)}(x-y)}{(x+2y)\cancel{(x-2y)}}$$

$$= \frac{x-y}{x+2y}$$

$$p) \quad \frac{8x^2 - 51x + 18}{8x^2 + 29x - 12} = \frac{(\cancel{8x-3})(x-\overset{3}{\underset{\uparrow}{6}})}{(\cancel{8x-3})(x+4)} = \frac{x-6}{x+4}$$

$$r) \quad \frac{\begin{array}{r} x(y+4) - 1(y+4) \\ xy + 4x - y - 4 \end{array}}{\begin{array}{r} xy + 4x - 4y - 16 \\ x(\underline{y+4}) - 4(\underline{y+4}) \end{array}} = \frac{\cancel{(y+4)}(x-1)}{(x-4)\cancel{(y+4)}} = \frac{x-1}{x-4}$$

$$n) \frac{5x^2 - 32x + 12}{4x^2 - 27x + 18} = \frac{(5x - 2)(x - 6)}{(4x - 3)(x - 6)} \\ = \frac{5x - 2}{4x - 3}$$

$$u) \frac{6 - x - 2x^2}{12 + 7x - 10x^2} = \frac{-(x^2 + x - 6)}{-(10x^2 - 7x - 12)}$$

$$= \frac{(2x - 3)(x + 2)}{(2x - 3)(5x + 4)} = \frac{x + 2}{5x + 4}$$

$$v) \frac{-(2x^2 - x - 15)}{x^2 - 10x + 21} = -\frac{(2x + 5)(x - 3)}{(x - 7)(x - 3)} \\ = -\frac{(2x + 5)}{x - 7}$$

$$\text{or } -\frac{2x + 5}{x - 7}$$

$$\begin{aligned}
 0) \quad \frac{7x^2 + 61x - 18}{7x^2 + 19x - 6} &= \frac{(\cancel{7x-2})(x+9)}{(\cancel{7x-2})(x+3)} \\
 &= \frac{x+9}{x+3}
 \end{aligned}$$

Multiplying $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} = \frac{ca}{db}$

p. 59

$$\begin{aligned}
 2. d) \quad \frac{8(x-3)}{3x+9} \cdot \frac{4x+12}{6x-18} &= \frac{16}{9} \\
 \frac{8(\cancel{x-3})}{3(\cancel{x+3})} \cdot \frac{4(\cancel{x+3})}{6(\cancel{x-3})} &= \frac{16}{9}
 \end{aligned}$$

$$\begin{aligned}
 r) \quad \frac{x^2 - xy - 6y^2}{x^2 - 4xy + 3y^2} \cdot \frac{y^2 - x^2}{x^2 + 3xy + 2y^2} &= -1 \\
 \frac{(\cancel{x-3y})(x+2y)}{(\cancel{x-3y})(x-y)} \cdot \frac{(\cancel{y-x})(y-x)}{(\cancel{x+2y})(x+y)} &= -1
 \end{aligned}$$

$$\frac{a}{\cancel{b}} \div \frac{c}{\cancel{d}} = \frac{a}{b} \times \frac{\cancel{d}}{c}$$

$$\begin{aligned} b &\neq 0 \\ d &\neq 0 \\ c &\neq 0 \end{aligned}$$

i) $\frac{4a^2 - ab - 5b^2}{(x+y)(a+b)} \div \frac{(4a-8b)}{4(a-2b)}$

$\frac{ax+by+ay+bx}{ax+ay+by+bx}$

$\frac{(4a-5b)(a+b)}{(x+y)(a+b)} \times \frac{1}{4(a-2b)} = \frac{4a-5b}{4(x+y)(a-2b)}$

$$2) \quad \frac{x(x^2-4)}{x^3-4x} \div \frac{x(4-x^2)}{4x-x^3} = \frac{\cancel{x} \cancel{(x+2)} \cancel{(x-2)}}{\cancel{x} \cancel{(1-x^2)}} \cdot \frac{\cancel{x} \cancel{(x^3-1)}}{\cancel{x} \cancel{(2+x)} \cancel{(2-x)}} = 1 \quad \checkmark$$

$$7) \quad \frac{x-8}{x-4} \div \left(\frac{\cancel{(x-8)}\cancel{(x-4)}}{x^2-12x+32} \cdot \frac{\cancel{x(x-8)}}{x^2-8x} \right)$$

$$\frac{\cancel{x-8}}{\cancel{x-4}} \cdot \frac{8(\cancel{x-4})}{(\cancel{x-8})(x-8)} = \frac{8}{x-8}$$

